1. There are seven dwarves. They each love to eat cupcakes, but hardly ever get the chance to. They come home one day to find that Snow White has baked seven cupcakes for them. In order to earn the cupcakes, Snow White has devised a game for the dwarves. She tells the dwarves:

“For this game, I am going to line you up single-file in height order with the tallest of you in the back of the line. You will all face the same direction so that only the tallest can see the rest of you, and the shortest will not be able to see anyone else. I will blindfold you and place hats on each of your heads. All the hats will be either red or black. Then I will take off your blindfolds. No one will know the color of his own hat or the color of the hats of the dwarves that are lined up behind you, but you will be able to see the colors of the hats of all the dwarves in the line in front of you. In turn, starting with the tallest dwarf (who is lined up last and can see all the hats), I will ask you what color your hat is. You may answer simply ‘red’ or ‘black’ and nothing more. If you get the answer right, you get a cupcake! If you get the answer wrong, you get nothing. I will continue this line of questioning all the way to the shortest dwarf in the front. Any leftover cupcakes I get to keep!”

She then gave the dwarves one hour to brainstorm how they can maximize the number of cupcakes they will earn in her game. At first the dwarves felt discouraged since Snow White had made it clear that complete silence was required during the game except for the answers ‘red’ or ‘black’ when they were each asked the color of their hats. They would also not be allowed to turn around or make any signals to each other of any kind. They dwarves all knew it was best not to cross Snow White by cheating or breaking her rules. A few of the dwarves decided that the best they can do is guess randomly at their own hat color when their turn came and hope they get lucky. Then, suddenly, Doc shouted, “Aha! I have a plan that will *guarantee* that we earn at least six cupcakes! One of us may be out of luck, but it’s the best we can do given the constraints of Snow White’s rules. Here’s how we do it…”

The dwarves worked together as a team and executed Doc’s plan successfully, earning at least six of the seven cupcakes. Afterward, Doc explained their strategy to Snow White who was so impressed by their cleverness that she baked seven more cupcakes for the dwarves that very night!

What was Doc’s plan? (If you have any further questions about the rules of the game, email Snow White at [cchung@ccsd.edu](mailto:cchung@ccsd.edu).) [Answer.](http://www.geocities.com/chung408/math2h/challengeAnswers.htm)